CHAPTER 11
LINEAR EQUATIONS IN ONE VARIABLE

One of the principal reasons for an intensive study of polynomials, grouping symbols, factoring, and fractions is to prepare for solving equations. The equation is perhaps the most important tool in algebra, and the more skillful the student becomes in working with equations, the greater will be his ease in solving problems.

Before learning to solve equations, it is necessary to become familiar with the words used in the discussion of them. An EQUATION is a statement that two expressions are equal in value. Thus,

$$4 + 5 = 9$$

and

$$A = lw$$

(Area of a rectangle = length x width)

are equations. The part to the left of the equality sign is called the LEFT MEMBER, or first member, of the equation. The part to the right is the RIGHT MEMBER, or second member, of the equation.

The members of an equation are sometimes thought of as corresponding to two weights that balance a scale. (See fig. 11-1.) This comparison is often helpful to students who are learning to solve equations. It is obvious, in

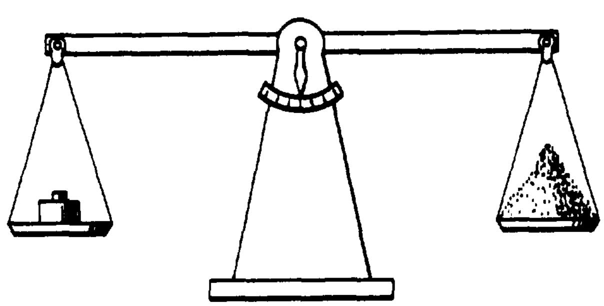

Figure 11-1. Equation compared to a balance scale.

the case of the scale, that any change made in one pan must be accompanied by an equal change in the other pan. Otherwise the scale will not balance. Operations on equations are based on the same principle. The members must be kept balanced or the equality is lost.

CONSTANTS AND VARIABLES

Expressions in algebra consist of constants and variables. A CONSTANT is a quantity whose value remains the same throughout a particular problem. A VARIABLE is a quantity whose value is free to vary.

There are two kinds of constants—fixed and arbitrary. Numbers such as 7, -3, 1/2, and π are examples of FIXED constants. Their values never change. In $5x + 7 = 0$, the numbers 0, 5, and 7, are fixed constants.

ARBITRARY constants can be assigned different values for different problems. Arbitrary constants are indicated by letters—quite often letters at the beginning of the alphabet such as a, b, c, and d. In

$$ax + b = 0,$$

the letters a and b represent arbitrary constants. The form $ax + b = 0$ represent many linear equations. If we give a and b particular values, say $a = 5$ and $b = 7$, then these constants become fixed, for this particular problem, and the equation becomes

$$5x + 7 = 0$$

A variable may have one value or it may have many values in a discussion. The letters at the end of the alphabet, such as x, y, z, and w, usually are used to represent variables. In $5x + 7$, the letter x is the variable. If $x = 1$, then

$$5x + 7 = 5 + 7 = 12$$

If $x = 2$, then

$$5x + 7 = 5(2) + 7 = 10 + 7 = 17$$

and so on for as many values of x as we desire to select.

If the expression $5x + 7$ is set equal to some particular number, say -23, then the resulting equality

$$5x + 7 = -23$$

holds true for just one value of x. The value is -6, since

$$5(-6) + 7 = -23$$

In an algebraic expression, terms that contain a variable are called VARIABLE TERMS. Terms that do not contain a variable are CONSTANT TERMS. The expression $5x + 7$ contains one variable term and one constant term. The variable term is 5x, while 7 is the constant term. In $ax + b$, ax is the variable term and b is the constant term.

A variable term often is designated by naming the variable it contains. In $5x + 7$, 5x is the x-term. In $ax + by$, ax is the x-term, while by is the y-term.

DEGREE OF AN EQUATION

The degree of an equation that has not more than one variable in each term is the exponent of the highest power to which that variable is raised in the equation. The equation

$$3x - 17 = 0$$

is a FIRST-DEGREE equation, since x is raised only to the first power.

An example of a SECOND-DEGREE equation is

$$5x^2 - 2x + 1 = 0.$$

The equation,

$$4x^3 - 7x^2 = 0,$$

is of the THIRD DEGREE.

The equation,

$$3x - 2y = 5$$

is of the first degree in two variables, x and y. When more than one variable appears in a term, as in $xy = 5$, it is necessary to add the exponents of the variables within a term to get the degree of the equation. Since $1 + 1 = 2$, the equation $xy = 5$ is of the second dgeree.

LINEAR EQUATIONS

Graphs are used in many different forms to give visual pictures of certain related facts. For example, they are used to show business trends, production output, continued individual attainment, and so forth. We find bar graphs, line graphs, circle graphs, and many other types, each of which is used for a particular need. In algebra, graphs are also used to give a visual picture containing a great deal of information about equations.

Sometimes many numerical values, when substituted for the variables of an equation, will satisfy the conditions of the equation. On a particular type of graph (which will be explained fully in chapter 12) several of these values are plotted (located), and when enough are plotted, a line is drawn through these points. For each particular equation a certain type of curve results. For equations in the first degree in one or two variables, the resulting shape of the "curve" is a straight line.. Thus, the name LINEAR EQUATION is derived. Equations of a higher degree form various other shapes. The name "linear equation" now applies to equations of the first degree, regardless of the number of variables they contain. Chapter 12 shows how an equation may be pictured on a graph. The purpose and value of graphing an equation will also be developed.

IDENTITIES

If a statement of equality involves one or more variables, it may be either an IDENTITY (identical equation) or a CONDITIONAL EQUATION. An identity is an equality that states a fact, such as the following examples:

1. $9 + 5 = 14$
2. $2n + 5n = 7n$
3. $6(x - 3) = 6x - 18$

Notice that equation 3 merely shows the factored form of $6x - 18$ and holds true when any value of x is substituted. For example, if $x = 5$, it becomes

$$6(5-3) = 6(5) - 18$$
$$6(2) = 30 - 18$$
$$12 = 12$$

If x assumes the negative value -10, this identity becomes

$$6(-10-3) = 6(-10)-18$$
$$6(-13) = -60-18$$
$$-78 = -78$$

An identity is established when both sides of the equality have been reduced to the same number or the same expression. When 5 is substituted for x, the value of either side of $6(x-3) = 6x - 18$ is 12. When -10 is substituted for x, the value on either side is -78. The fact that this equality is an identity can be shown also by factoring the right side so that the equality becomes

$$6(x-3) = 6(x-3)$$

The expressions on the two sides of the equality are identical.

CONDITIONAL EQUATIONS

A statement such as $2x-1 = 0$ is an equality only when x has one particular value. Such a statement is called a CONDITIONAL EQUATION, since it is true only under the condition that $x = 1/2$. Likewise, the equation $y - 7 = 8$ holds true only if $y = 15$.

The value of the variable for which an equation in one variable holds true is a ROOT, or SOLUTION, of the equation. When we speak of solving equations in algebra, we refer to conditional equations. The solution of a conditional equation can be verified by substituting for the variable its value, as determined by the solution.

The solution is correct if the equality reduces to an identity. For example, if 1/2 is substituted for x in $2x - 1 = 0$, the result is

$$2\left(\frac{1}{2}\right) - 1 = 0$$
$$1 - 1 = 0$$
$$0 = 0 \quad \text{(an identity)}$$

The identity is established for $x = \frac{1}{2}$, since the value of each side of the equality reduces to zero.

SOLVING LINEAR EQUATIONS

Solving a linear equation in one variable means finding the value of the variable that makes the equation true. For example, 11 is the SOLUTION of $x - 7 = 4$, since $11 - 7 = 4$. The number 11 is said to SATISFY the equation. Basically, the operation used in solving equations is to manipulate both members, by addition, subtraction, multiplication, or division until the value of the variable becomes apparent. This manipulation may be accomplished in a straightforward manner by use of the axioms outlined in chapter 3 of this course. These axioms may be summed up in the following rule: If both members of an equation are increased, decreased, multiplied, or divided by the same number, or by equal numbers, the results will be equal. (Division by zero is excluded.)

As mentioned earlier, an equation may be compared to a balance. What is done to one member must also be done to the other to maintain a balance. An equation must always be kept in balance or the equality is lost. We use the above rule to remove or adjust terms and coefficients until the value of the variable is discovered. Some examples of equations solved by means of the four operations mentioned in the rule are given in the following paragraphs.

ADDITION

Find the value of x in the equation

$$x - 3 = 12$$

As in any equation, we must isolate the variable on either the right or left side. In this problem, we leave the variable on the left and perform the following steps:

1. Add 3 to both members of the equation, as follows:

$$x - 3 + 3 = 12 + 3$$

In effect, we are "undoing" the subtraction indicated by the expression $x - 3$, for the purpose of isolating x in the left member.

2. Combining terms, we have

$$x = 15$$

SUBTRACTION

Find the value of x in the equation

$$x + 14 = 24$$

1. Subtract 14 from each member. In effect, this undoes the addition indicated in the expression $x + 14$.

$$x + 14 - 14 = 24 - 14$$

2. Combining terms, we have

$$x = 10$$

MULTIPLICATION

Find the value of y in the equation

$$\frac{y}{5} = 10$$

1. The only way to remove the 5 so that the y can be isolated is to undo the indicated division. Thus we use the inverse of division, which is multiplication. Multiplying both members by 5, we have the following:

$$5\left(\frac{y}{5}\right) = 5(10)$$

2. Performing the indicated multiplications, we have

$$y = 50$$

DIVISION

Find the value of x in the equation

$$3x = 15$$

1. The multiplier 3 may be removed from the x by dividing the left member by 3. This must be balanced by dividing the right member by 3 also, as follows:

$$\frac{3x}{3} = \frac{15}{3}$$

2. Performing the indicated divisions, we have

$$x = 5$$

Practice problems. Solve the following equations:

1. $m + 2 = 8$	4. $\frac{x}{14} = 2$
2. $x - 5 = 11$	5. $2n = 5$
3. $6x = -48$	6. $\frac{1}{6}y = 6$

Answers:

1. $m = 6$	4. $x = 28$
2. $x = 16$	5. $n = 2\frac{1}{2}$
3. $x = -8$	6. $y = 36$

SOLUTIONS REQUIRING MORE THAN ONE OPERATION

Most equations involve more steps in their solutions than the simple equations already described, but the basic operations remain unchanged. If the basic axioms are kept well in mind, these more complicated equations will not become too difficult. Equations may require one or all of the basic operations before a solution can be obtained.

Subtraction and Division

Find the value of x in the following equation:

$$2x + 4 = 16$$

1. The term containing x is isolated on the left by subtracting 4 from the left member. This operation must be balanced by also subtracting 4 from the right member, as follows:

$$2x + 4 - 4 = 16 - 4$$

2. Performing the indicated operations, we have

$$2x = 12$$

3. The multiplier 2 is removed from the x by dividing both sides of the equation by 2, as follows:

$$\frac{2x}{2} = \frac{12}{2}$$

$$x = 6$$

Addition, Multiplication, and Division

Find the value of y in the following equation:

$$\frac{3y}{2} - 4 = 11$$

1. Isolate the term containing y on the left by adding 4 to both sides, as follows:

$$\frac{3y}{2} - 4 + 4 = 11 + 4$$

$$\frac{3y}{2} = 15$$

2. Since the 2 will not divide the 3 exactly, multiply the left member by 2 in order to eliminate the fraction. This operation must be balanced by multiplying the right member by 2, as follows:

$$2\left(\frac{3y}{2}\right) = 2(15)$$

$$3y = 30$$

3. Divide both members by 3, in order to isolate the y in the left member, as follows:

$$\frac{3y}{3} = \frac{30}{3}$$

$$y = 10$$

Equations Having the Variable in More Than One Term

Find the value of x in the following equation:

$$\frac{3x}{4} + x = 12 - x$$

1. Rewrite the equation with no terms containing the variable in the right member. This requires adding x to the right member to eliminate the -x term, and balance requires that we also add x to the left member, as follows:

$$\frac{3x}{4} + x + x = 12 - x + x$$

$$\frac{3x}{4} + 2x = 12$$

2. Since the 4 will not divide the 3 exactly, it is necessary to multiply the first term by 4 to eliminate the fraction. However, notice that this multiplication cannot be performed on the first term only; any multiplier which is introduced for simplification purposes must be applied to the entire equation. Thus each term in the equation is multiplied by 4, as follows:

$$4\left(\frac{3x}{4}\right) + 4(2x) = 4(12)$$

$$3x + 8x = 48$$

3. Add the terms containing x and then divide both sides by 11 to isolate the x in the left member, as follows:

$$11x = 48$$

$$x = \frac{48}{11}$$

$$= 4\frac{4}{11}$$

Practice problems. Solve each of the following equations:

1. $x - 1 = \frac{1}{2}$
2. $\frac{y}{3} + y = 8$
3. $\frac{x}{4} + 3x = 7$
4. $4 - 7x = 9 - 8x$
5. $\frac{y}{2} + 6y = 13$
6. $\frac{1}{2}x - 2x = 25 + x$

Answers:

1. $x = 3/2$
2. $y = 6$
3. $x = 28/13$
4. $x = 5$
5. $y = 2$
6. $x = -10$

EQUATIONS WITH LITERAL COEFFICIENTS

As stated earlier, the first letters of the alphabet usually represent known quantities (constants), and the last letters represent unknown quantities (variables). Thus, we usually solve for x, y, or z.

An equation such as

$$ax - 8 = bx - 5$$

has letters as coefficients. Equations with literal coefficients are solved in the same way as equations with numerical coefficients, except that when an operation cannot actually be performed, it merely is indicated.

In solving for x in the equation

$$ax - 8 = bx - 5$$

subtract bx from both members and add 8 to both members. The result is

$$ax - bx = 8 - 5$$

Since the subtraction on the left side cannot actually be performed, it is indicated. The quantity, a - b, is the coefficient of x when terms are collected. The equation takes the form

$$(a-b)\,x = 3$$

Now divide both sides of the equation by a-b. Again the result can be only indicated. The solution of the equation is

$$x = \frac{3}{a-b}$$

In solving for y in the equation

$$ay + b = 4$$

subtract b from both members as follows:

$$ay = 4 - b$$

Dividing both members by a, the solution is

$$y = \frac{4-b}{a}$$

Practice problems. Solve for x in each of the following:

1. $3 + x = b$
2. $4x = 8 + t$
3. $3x + 6m = 7m$
4. $ax - 2(x + b) = 3a$

Answers:

1. $x = b - 3$
2. $x = \frac{8 + t}{4}$
3. $x = \frac{m}{3}$
4. $x = \frac{3a + 2b}{a - 2}$

REMOVING SIGNS OF GROUPING

If signs of grouping appear in an equation they should be removed in the manner indicated in chapter 9 of this course. For example, solve the equation

$$5 = 24 - [x-12(x-2) - 6(x-2)]$$

Notice that the same expression, x-2, occurs in both parentheses. By combining the terms containing (x-2), the equation becomes

$$5 = 24 - [x-18(x-2)]$$

Next, remove the parentheses and then the bracket, obtaining

$$\begin{aligned} 5 &= 24 - [x-18x + 36] \\ &= 24 - [36 - 17x] \\ &= 24 - 36 + 17x \\ &= -12 + 17x \end{aligned}$$

Subtracting 17x from both members and then subtracting 5 from both members, we have

$$-17x = -12 - 5$$
$$-17x = -17$$

Divide both members by -17. The solution is

$$x = 1$$

EQUATIONS CONTAINING FRACTIONS

To solve for x in an equation such as

$$\frac{2x}{3} + \frac{x}{12} - 1 = \frac{1}{4} + \frac{x}{2}$$

first clear the equation of fractions. To do this, find the least common denominator of the fractions. Then multiply both sides of the equation by the LCD.

The least common denominator of 3, 12, 4, and 2 is 12. Multiply both sides of the equation by 12. The resulting equation is

$$8x + x - 12 = 3 + 6x$$

Subtract 6x from both members, add 12 to both members, and collect like terms as follows:

$$9x - 6x = 12 + 3$$
$$3x = 15$$

The solution is

$$x = 5$$

To prove that x = 5 is the correct solution, substitute 5 for x in the original equation and show that both sides of the equation reduce to the same value. The result of substitution is

$$\frac{2(5)}{3} + \frac{5}{12} - 1 = \frac{1}{4} + \frac{5}{2}$$

In establishing an identity, the two sides of the equality are treated separately, and the operations are performed as indicated. Sometimes, as here, fractions occur on both sides of the equality, and it is desirable to find the least common denominator for more than one set of fractions. The same denominator could be used on both sides of the equality, but this might make some of the terms of the fractions larger than necessary.

Proceeding in establishing the identity for x = 5 in the foregoing equation we obtain

$$\frac{10}{3} + \frac{5}{12} - \frac{3}{3} = \frac{1}{4} + \frac{10}{4}$$

$$\frac{7}{3} + \frac{5}{12} = \frac{11}{4}$$

$$\frac{28}{12} + \frac{5}{12} = \frac{11}{4}$$

$$\frac{33}{12} = \frac{11}{4}$$

$$\frac{11}{4} = \frac{11}{4}$$

Each member of the equality has the value 11/4 when x = 5. The fact that the equation becomes an identity when x is replaced by 5 proves that x = 5 is the solution.

Practice problems. Solve each of the following equations:

1. $\frac{x}{4} - 2 = \frac{x}{6}$
2. $\frac{1}{2} - \frac{1}{v} = \frac{1}{3}$
3. $\frac{y}{2} - \frac{y}{3} = 5$
4. $\frac{3}{4x} = 6$

Answers:

1. x = 24
2. v = 6
3. y = 30
4. x = 1/8

GENERAL FORM OF A LINEAR EQUATION

The expression GENERAL FORM, in mathematics, implies a form to which all expressions or equations of a certain type can be reduced. The only possible terms in a linear equation in one variable are the first-degree term and the constant term. Therefore, the general form of a linear equation in one variable is

$$ax + b = 0$$

By selecting various values for a and b, this form can represent any linear equation in one variable after such an equation has been simplified. For example, if a = 7 and b = 5, ax + b = 0 represents the numerical equation

$$7x + 5 = 0$$

If a = 2m - n and b = p - q, then ax + b = 0 represents the literal equation

$$(2m-n)x + p - q = 0$$

This equation is solved as follows:

$$(2m-n)x + (p-q) - (p-q) = 0 - (p-q)$$

$$(2m-n)x = 0 - (p-q)$$

$$x = \frac{-(p-q)}{2m-n} = \frac{q-p}{2m-n}$$

USING EQUATIONS TO SOLVE PROBLEMS

To solve a problem, we first translate the numerical sense of the problem into an equation. To see how this is accomplished, consider the following examples and their solutions.

EXAMPLE 1: Together Smith and Jones have \$120. Jones has 5 times as much as Smith. How much has Smith?

SOLUTION:

Step 1. Get the problem clearly in mind. There are two parts to each problem—what is given (the facts) and what we want to know (the question). In this problem we know that Jones has 5 times as much as Smith and together they have \$120. We want to know how much Smith has.

Step 2. Express the unknown as a letter. Usually we express the unknown or number we know the least about as a letter (conventionally we use x). Here we know the least about Smith's money. Let x represent the number of dollars Smith has.

Step 3. Express the other facts in terms of the unknown. If x is the number of dollars Smith has and Jones has 5 times as much, then 5x is the number of dollars Jones has.

Step 4. Express the facts as an equation. The problem will express or imply a relation between the expressions in steps 2 and 3. Smith's dollars plus Jones' dollars equal $120. Translating this statement into algebraic symbols, we have

$$x + 5x = 120$$

Solving the equation for x,

$$6x = 120$$
$$x = 20$$

Thus Smith has $20.

Step 5. Check: See if the solution satisfies the original statement of the problem. Smith and Jones have $120.

$20	+	$100	= $120
(Smith's money)		(Jones' money)	

EXAMPLE 2: Brown can do a piece of work in 5 hr. If Olsen can do it in 4 hr how long will it take them to do the work together?

SOLUTION:

Step 1. Given: Brown could do the work in 5 hr. Olsen could do it in 4 hours.

Unknown: How long it takes them to do the work together.

Step 2. Let x represent the time it takes them to do the work together.

Step 3. Then $\frac{1}{x}$ is the amount they do together in 1 hr. Also, in 1 hour Brown does $\frac{1}{5}$ of the work and Olsen does $\frac{1}{4}$ of the work.

Step 4. The amount done in 1 hr is equal to the part of the work done by Brown in 1 hr plus that done by Olsen in 1 hr.

$$\frac{1}{x} = \frac{1}{5} + \frac{1}{4}$$

Solving the equation,

$$20x\left(\frac{1}{x}\right) = 20x\left(\frac{1}{5}\right) + 20x\left(\frac{1}{4}\right)$$
$$20 = 4x + 5x$$
$$20 = 9x$$
$$\frac{20}{9} = x, \text{ or } x = 2\tfrac{2}{9} \text{ hours}$$

They complete the work together in $2\frac{2}{9}$ hours.

Step 5. Check: $2\frac{2}{9} \times \frac{1}{5}$ = amount Brown does

$2\frac{2}{9} \times \frac{1}{4}$ = amount Olsen does

$$\left(\frac{20}{9} \times \frac{1}{5}\right) + \left(\frac{20}{9} \times \frac{1}{4}\right) = \frac{4}{9} + \frac{5}{9} = \frac{9}{9}$$

Practice problems. Use a linear equation in one variable to solve each of the following problems:

1. Find three numbers such that the second is twice the first and the third is three times as large as the first. Their sum is 180.

2. A seaman drew $75.00 pay in dollar bills and five-dollar bills. The number of dollar bills was three more than the number of five-dollar bills. How many of each kind did he draw? (Hint: If x is the number of five-dollar bills, then 5x is the number of dollars they represent.)

3. Airman A can complete a maintenance task in 4 hr. Airman B requires only 3 hr to do the same work. If they work together, how long should it take them to complete the job?

Answers:

1. First number is 30.
   Second number is 60.
   Third number is 90.

2. Number of five-dollar bills is 12.
   Number of one-dollar bills is 15.

3. $1\frac{5}{7}$ hr.

INEQUALITIES

Modern mathematical thought gives considerable emphasis to the concept of inequality. A meaningful comparison between two quantities can be set up if they are related in some way, even though the relationship may not be one of equality.

The expression "number sentence" is often used to describe a general relationship which may be either an equality or an inequality. If the number sentence states an equality, it is an EQUATION; if it states an inequality, it is an INEQUATION.

ORDER PROPERTIES OF REAL NUMBERS

The idea of order, or relative rank according to size, is based upon two intuitive concepts: "greater than" and "less than." Mathematicians use the symbol $>$ to represent "greater than" and the symbol $<$ to represent "less than." For example, the inequation stating that 7 is greater than 5 is written in symbols as follows:

$$7 > 5$$

The inequation stating that x is less than 10 is written as follows:

$$x < 10$$

A "solution" of an inequation involving a variable is any number which may be substituted for the variable without changing the relationship between the left member and the right member. For example, the inequation $x < 10$ has many solutions. All negative numbers zero, and all positive numbers less than 10, may be substituted for x successfully. These solutions comprise a set of numbers, called the SOLUTION SET.

The SENSE of an inequality refers to the direction in which the inequality symbol points. For example, the following two inequalities have opposite sense:

$$7 > 5$$
$$10 < 12$$

PROPERTIES OF INEQUALITIES

Inequations may be manipulated in accordance with specific operational rules, in a manner similar to that used with equations.

Addition

The rule for addition is as follows: If the same quantity is added to both members of an inequation, the result is an inequation having the same sense as the original inequation. The following examples illustrate this:

1. $$5 < 8$$
$$5 + 2 < 8 + 2$$
$$7 < 10$$

The addition of 2 to both members does not change the sense of the inequation.

2. $$5 < 8$$
$$5 + (-3) < 8 + (-3)$$
$$2 < 5$$

The addition of -3 to both members does not change the sense of the inequation.

Addition of the same quantity to both members is a useful method for solving inequations. In the following example, 2 is added to both members in order to isolate the x term on the left:

$$x - 2 > 6$$
$$x - 2 + 2 > 6 + 2$$
$$x > 8$$

Multiplication

The rule for multiplication is as follows: If both members of an inequation are multiplied by the same positive quantity, the sense of the resulting inequation is the same as that of the original inequation. This is illustrated as follows:

1. $$-3 < -2$$
$$2(-3) < 2(-2)$$
$$-6 < -4$$

Multiplication of both members by 2 does not change the sense of th inequation.

2. $$10 < 12$$
$$\frac{1}{2}(10) < \frac{1}{2}(12)$$
$$5 < 6$$

Multiplication of both members by 1/2 does not change the sense of the inequation.

Notice that example 2 illustrates division of both members by 2. Since any division can be rewritten as multiplication by a fraction, the multiplication rule is applicable to both multiplication and division.

Multiplication is used to simplify the solution of inequations such as the following:

$$\frac{x}{3} > 2$$

Multiply both members by 3:

$$3\left(\frac{x}{3}\right) > 3(2)$$

$$x > 6$$

Sense Reversal

If both sides of an inequation are multiplied or divided by the same negative number, the sense of the resulting inequation is reversed. This is illustrated as follows:

1. $$-4 < -2$$
$$(-3)(-4) > (-3)(-2)$$
$$12 > 6$$

2. $$7 > 5$$
$$(-2)(7) < (-2)(5)$$
$$-14 < -10$$

Sense reversal is useful in the solution of an inequation in which the variable is preceded by a negative sign, as follows:

$$2 - x < 4$$

Add -2 to both members to isolate the x term:

$$2 - x - 2 < 4 - 2$$

$$-x < 2$$

Multiply both members by -1:

$$x > -2$$

Practice problems. Solve each of the following inequations:

1. $x + 2 > 3$	3. $3 - x < 6$
2. $\frac{y}{3} - 1 < 2$	4. $4y > 8$

Answers:

1. $x > 1$	3. $x > -3$
2. $y < 9$	4. $y > 2$

GRAPHING INEQUALITIES

An inequation such as $x > 2$ can be graphed on a number line, as shown in figure 11-2.

The heavy line in figure 11-2 contains all values of x which comprise the solution set. Notice that this line continues indefinitely in the positive direction, as indicated by the arrow head. Notice also that the point representing $x = 2$ is designated by a circle. This signifies that the solution set does not contain the number 2.

Figure 11-3 is a graph of the inequation $x^2 > 4$. Since the square of any number greater than 2 is greater than 4, the solution set contains all values of x greater than 2. Furthermore, the solution set contains all values of x less than -2. This is because the square of any negative number smaller than -2 is a positive number greater than 4.

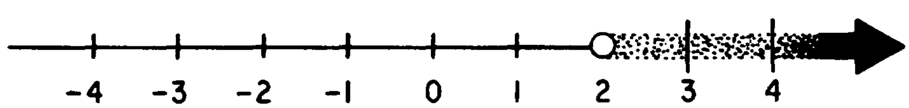

Figure 11-2.—Graph of the inequation $x > 2$.

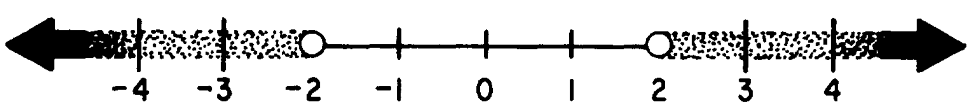

Figure 11-3.—Graph of $x^2 > 4$.